Three-Loop Predictions for the Light Higgs Mass in the MSSM

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The Minimal Supersymmetric Extension of the Standard Model (MSSM) features a light Higgs boson, the mass M_h of which is predicted by the theory. Given that the LHC will be able to measure the mass of a light Higgs with great accuracy, a precise theoretical calculation of M_h yields an important test of the MSSM. In order to deliver this precision, we present three-loop radiative corrections of $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$ and provide a computer code that combines our results with corrections to M_h at lower loop orders that are available in the literature.

1 Introduction

The Higgs sector of the Minimal Supersymmetric Extension of the Standard Model (MSSM) consists of a two-Higgs doublet model, which is tightly constrained by Supersymmetry. In particular, the quartic terms of the Higgs potential are completely fixed by the gauge couplings. Thus, it is possible to describe the MSSM Higgs sector through only two new (with respect to the Standard Model) parameters, which are usually taken to be the mass M_A of the pseudoscalar Higgs and the ratio $\tan\beta = \frac{v_2}{v_1}$ of the vacuum expectation values of the Higgs doublets. In particular, M_h, the mass of the light scalar Higgs boson, can be predicted, and at the tree-level only these two parameters enter the prediction, leading to an upper bound of $M_h \leq M_Z$. However, M_h is sensitive to virtual corrections to the Higgs propagator that shift this upper bound significantly. These virtual corrections depend on all the Supersymmetry breaking parameters. This sensitivity to virtual corrections, combined with the great precision with which the Large Hadron Collider (LHC) will be able to measure the mass of a light Higgs, allows M_h to be used as a precision observable to test supersymmetric models – assuming that the theoretical uncertainties are sufficiently small and under control.

Consequently, the one- and two-loop corrections to M_h have been studied extensively in the literature (see, for example [1, 2, 3, 4, 5, 6, 7, 8]). The remaining uncertainty has been estimated to be about $3 - 5\,\text{GeV}$ [10, 9]. Recently, also three-loop corrections have become available. The leading- and next-to-leading terms in $\ln(M_{SUSY}/M_t)$, where M_{SUSY} is the typical scale of SUSY particle masses, have been obtained in [11]. Motivated by the observation that the contributions from loops of top quarks and their superpartners, the stops, are dominant at the one- and two-loop level, we have calculated three-loop SUSY-QCD corrections to these diagrams. These corrections are of $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$, where α_t is the coupling of the Higgs to the top quarks. A first result has been obtained in [12]. There, we assumed that all the superpartners had approximately the same mass. This restriction has been dropped recently in [13].
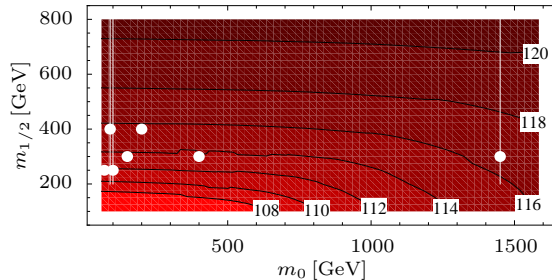
Figure 1: Prediction for the value of M_h (in GeV) for MSUGRA scenario with $\tan\beta = 10$, $A_0 = 0$, as evaluated by H3M. The white lines and points indicate the benchmark scenarios of [19].

2   Organisation of the Calculation

A major difficulty in obtaining the results of [13] was the presence of many different mass scales – the masses m_t of the top quark, $m_{\tilde{g}}$ of the gluino, $m_{\tilde{t}_{1,2}}$ of the stops and $m_{\tilde{q}}$ of the partners of the light quarks – in the three-loop propagator diagrams. Assuming that there is a distinct hierarchy between these masses, they can be disentangled by the method of asymptotic expansions [14], yielding an expansion of the diagrams in small mass ratios and logarithms of mass ratios. Working in the effective potential approximation, we set the external momentum flowing through the Higgs propagator to zero and are left with tadpole integrals with a single mass scale, which are known and implemented in the FORM [15] program MATAD [16].

However, as the masses of the superpartners are not known, it is not clear which hierarchy one should assume. We solve this by computing the diagrams for many different hierarchies. Then, when given a point in the MSSM parameter space, we choose whichever hierarchy fits best and evaluate M_h using the calculation in the chosen hierarchy. To choose the best hierarchy and to estimate the error introduced by the asymptotic expansion, we compare, at the two-loop level, our expanded result with the result of [7], which contains the full mass dependence.

For convenience, we have written the MATHEMATICA package H3M [17], which automatically performs the choice of the best fitting hierarchy and provides a SUSY LES HOUCHES interface to our calculation. This allows to perform parameter scans as in Fig. 1. In order to get a state-of-the art prediction for M_h, we include all available contributions to M_h at the one- and two-loop level that are implemented in FEYNHIGGS [18]. For details on the usage and inner workings of the program, we refer to [13].

3   Estimating the Theoretical Uncertainty

We observe that the dependence of M_h on the renormalisation prescription, which is often used as a guesstimate for the uncertainty due to unknown higher order corrections, reduces drastically when one goes from two to three loops. But since we also find that the size of the three-loop corrections can be of the order of one to two GeV, which is rather large given that the two-loop corrections are only about a factor of two larger, we prefer to be conservative in our estimation of the theoretical uncertainty. Assuming a geometric progression of the perturbative series, we get for MSUGRA scenarios an uncertainty due to missing higher order corrections of $100\,\text{MeV}$ to $1\,\text{GeV}$, depending on the value of $m_{1/2}$. The parametric uncertainty due to α_s, m_t

and $m_{\tilde{t}_{1,2}}$ is of the same order of magnitude. The uncertainty introduced by the expansion in mass ratios amounts to at most $100\,\mathrm{MeV}$ [13].

4 Conclusions

We present a calculation of the $\mathcal{O}\left(\alpha_t\alpha_s^2\right)$ corrections to M_h, shifting the value of M_h by about $1\,\mathrm{GeV}$. We provide a computer code combining our results with corrections from lower loop orders, thus enabling a state-of-the-art prediction of M_h. Our calculation lowers the theoretical uncertainty due to missing higher orders to the same magnitude as the parametric uncertainty.

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